SIX ORIGINAL PROBLEMS.

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Remark. — If the gravitating force requires time for its transmission through space, it can be easily proved, by the well known principle of the parallelogram of velocities and forces, that every planetary body must be accelerated in its orbit, and, consequently, must recede from the gravitating center in an outward spiral path, unless such propulsion is counteracted by a resisting medium.

Problem I. — Assuming that every particle of matter in the universe transmits its gravitating force to every other particle with the velocity of the waves of light,* heat, electricity, and other solar radiations, that is, at the rate of 185420 miles per second; prove that the propelling and resisting forces of the planets, in circular orbits, must be directly as the masses, and inversely as the fifth powers of the square roots of the respective distances from the sun.

Problem II. — When the two antagonistic forces, in circular orbits, are equal, what must be the law of density of the resisting etherial medium, expressed in terms of the sun's distance?

Problem III. — Given the earth's mean radius equal to 3955.94943182 miles; distance from the sun = 91430000 miles; sun's mass (that of the earth = 1) = 314760; earth's density (that of water = 1) = 5.6604; weight of a cubic inch of distilled water = 252.458 grains avoirdupois; one pound, when freed from the effects of centrifugal force of the earth's axial rotation, and from atmospheric inflaences, = 7000 grains; the velocity of gravity as in problem I, — to find the intensity of the orbital and resisting forces, expressed in pounds weight, to permanently maintain the earth in a circular orbit.

Problem IV. — Assuming the velocity of gravity as in problem I, and the equality of the two forces as in problem II,—prove that the aberrating forces, in any two points of an elliptic orbit, vary inversely as the cubes of their distances from the center of gravity, situated in the lower focus of the ellipse.

*Laplace concludes, from a consideration of the moon's secular equation, that, if gravitation is produced by the impulse of a fluid directed towards the center of the attracting body, the velocity of the gravitating fluid must be, at least, *a huudred millions of times greater than that of light.*—See Mecanique Celeste, Bowditch's translation, Vol. IV, p. 645.—Ed.

Problem V. — Assuming the same conditions as in problem IV, prove that in any two points of an elliptic orbit, the resistance will vary directly as the distance to the upper focus, and inversely as the fifth power of the square root of the distance to the lower focus.

Problem VI.—If the two forces are equal in circular orbits; and if a is equal to the semi-major axis of the earth's orbit; and v, g and r are respectively equal to the orbital velocity, intensity of the orbital force of gravity, and the etherial resistance of the earth at its mean distance from the sun; and if $a' =$ the semi-major axis, and $b' =$ the semi-minor axis of any elliptic orbit,—prove that one of the values of x, in the equation,

$$x^3 - 4a'x^2 + 4a'^2x - v^2g^2b'^2a'^2 \div r^2 = 0,$$

will be equal to the length of the radius vector to that point in the ellipse, where the accelerating force will be exactly balanced by the resisting force.

Note by Prof. M. C. Stevens.—In the article on Repetends published in Nos. 1 and 2 of Vol. I, it is stated, p. 25, lines 7 to 10, that "the operation may be very much abbreviated by taking advantage of the well-known property of repetends, that after one half of the figures are obtained, the second half may be found by subtracting each figure of the first half successively from 9;" and in demonstrating this property, on p. 27, it is assumed, if $1 \div d$ be a fraction that reduces to a repetend, that in the reduction by division there will occur the remainder $d - 1$.

It should have been stated that this property applies only to such fractions reducing to pure repetends as have for denominators *prime numbers.*

It is therefore evident that the property is not applicable to the repetends resulting from $\frac{1}{27}$, $\frac{1}{63}$, $\frac{1}{81}$, &c.

The restriction is implied in the proof, since the remainder $d - 1$ only occurs in case the denominator is a prime number.

Note by the Editor. — The solution of problem 105, given at p. 127, is defective, because the conclusion is virtually assumed by placing $q - p = m - A + B - C + D -$ &c. If we write instead, $p - q = -m + A - B + C - D$ &c., we shall prove, in a similar manner, that $p > q$.

We subjoin Dr. Nelson's solution of 105, which is entirely rigorous.

"The number of odd selections (1 at a time, 3 at a time, &c.) out of n shot

$$= S = n + \frac{n(n-1)(n-2)}{3!} + \frac{n(n-1)(n-2)(n-3)(n-4)}{5!} + \text{\&c.}$$